NAG Toolbox for MATLAB

f07ba

1 Purpose

f07ba computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n band matrix, with k_l subdiagonals and k_u superdiagonals, and X and B are n by r matrices.

2 Syntax

```
[ab, ipiv, b, info] = f07ba(kl, ku, ab, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07ba uses the LU decomposition with partial pivoting and row interchanges to factor A as $A = PLU$, where P is a permutation matrix, L is a product of permutation and unit lower triangular matrices with k_l subdiagonals, and U is upper triangular with $(k_l + k_u)$ superdiagonals. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **kl – int32 scalar**

k_l, the number of subdiagonals within the band of the matrix A.

Constraint: $\mathbf{kl} \geq 0$.

2: **ku – int32 scalar**

k_u, the number of superdiagonals within the band of the matrix A.

Constraint: $\mathbf{ku} \geq 0$.

3: **ab(ldab,*) – double array**

The first dimension of the array **ab** must be at least $2 \times \mathbf{kl} + \mathbf{ku} + 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n coefficient matrix A.

The matrix is stored in rows $k_l + 1$ to $2k_l + k_u + 1$; the first k_l rows need not be set, more precisely, the element A_{ij} must be stored in

$A_{ij} = \mathbf{ab}(k_l + k_u + 1 + i - j, j) \quad \text{for } \max(1, j - k_u) \leq i \leq \min(n, j + k_l)$.

See Section 8 for further details.

4: **b(ldb,$*$) – double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **ab**.

n, the number of linear equations, i.e., the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, ldb

5.4 Output Parameters

1: **ab(ldab,$*$) – double array**

The first dimension of the array **ab** must be at least $2 \times \mathbf{kl} + \mathbf{ku} + 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $\mathbf{info} \geq 0$, **ab** contains details of the factorization.

The upper triangular band matrix U, with $k_l + k_u$ superdiagonals, is stored in rows 1 to $k_l + k_u + 1$ of the array, and the multipliers used to form the matrix L are stored in rows $k_l + k_u + 2$ to $2k_l + k_u + 1$.

2: **ipiv($*$) – int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

If $\mathbf{info} \geq 0$, the pivot indices that define the permutation matrix P; at the ith step row i of the matrix was interchanged with row $\mathbf{ipiv}(i)$. $\mathbf{ipiv}(i) = i$ indicates a row interchange was not required.

3: **b(ldb,$*$) – double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

If $\mathbf{info} = 0$, the n by r solution matrix X.

4: **info – int32 scalar**

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **n**, 2: **kl**, 3: **ku**, 4: **nrhs_p**, 5: **ab**, 6: **ldab**, 7: **ipiv**, 8: **b**, 9: **ldb**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, u_{ii} is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* 1999 for further details.

Following the use of f07ba, f07bg can be used to estimate the condition number of A and f07bh can be used to obtain approximate error bounds. Alternatives to f07ba, which return condition and error estimates directly are f04bb and f07bb.

8 Further Comments

The band storage scheme for the array **ab** is illustrated by the following example, when $n = 6$, $k_l = 1$, and $k_u = 2$. Storage of the band matrix A in the array **ab**:

$$\begin{array}{cccccc} * & * & * & + & + & + \\ * & * & a_{13} & a_{24} & a_{35} & a_{46} \\ * & a_{12} & a_{23} & a_{34} & a_{45} & a_{56} \\ a_{11} & a_{22} & a_{33} & a_{44} & a_{55} & a_{66} \\ a_{21} & a_{32} & a_{43} & a_{54} & a_{65} & * \end{array}$$

Array elements marked $*$ need not be set and are not referenced by the function. Array elements marked $+$ need not be set, but are defined on exit from the function and contain the elements u_{14}, u_{25} and u_{36}.

The total number of floating-point operations required to solve the equations $AX = B$ depends upon the pivoting required, but if $n \gg k_l + k_u$ then it is approximately bounded by $O(nk_l(k_l + k_u))$ for the factorization and $O(n(2k_l + k_u)r)$ for the solution following the factorization.

The complex analogue of this function is f07bn.

9 Example

```
kl = int32(1);
ku = int32(2);
ab = [0, 0, 0, 0;
     0, 0, -3.66, -2.13;
     0, 2.54, -2.73, 4.07;
     -0.23, 2.46, 2.46, -3.82;
     -6.98, 2.56, -4.78, 0];
b = [4.42;
     27.13;
     -6.14;
     10.5];
[abOut, ipiv, bOut, info] = f07ba(kl, ku, ab, b)
```

```
abOut =
         0         0         0   -2.1300
         0         0   -2.7300    4.0700
         0    2.4600    2.4600   -3.8391
   -6.9800    2.5600   -5.9329   -0.7269
    0.0330    0.9605    0.8057         0
ipiv =
           2
           3
           3
           4
bOut =
   -2.0000
    3.0000
    1.0000
   -4.0000
info =
           0
```